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On the Measurement of a Meteor Trail on a Photographic Plate.

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1. On August 22, 1907, Mr. F. W. Longbottom, of Haslemere, Queen's Park, Chester, sent me the print of a nebula photograph across which a meteor had flashed, asking for advice about the measurement. I have not seen any note on the easiest way of utilising such trails, which have already appeared in a good many photographs, and will doubtless occur in a good many more; and the following method may be found useful in other cases.

2. The rule is simply—measure on any convenient uniform scale, and by any convenient method, the distances of any known stars from the trail. (Three stars are theoretically sufficient, but more will give better results.) These measures will give the desired information.

3. The information obtainable is the position of the pole of the trail. The trail is assumed to be part of a straight line on the plate or of a great circle on the sphere. We cannot determine the radiant from a single trail, only the great circle on which the radiant lies; and this is conveniently specified by its pole, in R.A. and Decl. A_m and D_m say. If we can find other trails belonging to the same radiant, their poles will lie on a great circle of which the radiant is the pole.

4. Let (A, D) be the co-ordinates of the plate centre; (ξ, η) the standard co-ordinates of any point on the plate, *expressed in parts of the radius*, the R.A. and Decl. being (α, δ). If (α, δ) lies on the meteor trail, it is 90° distant from (A_m, D_m), and hence

$$\sin\delta\sin D_m + \cos\delta\cos D_m\cos(\alpha - A_m) = 0 \quad . \qquad (1)$$

Now (α, δ) may be expressed in terms of (ξ, η) by the following formulæ (*Monthly Notices*, lxiv. p. 648):—

$$\frac{\cos\delta\sin(\alpha - A)}{\xi} = \frac{\sin\delta}{\sin D + \eta\cos D} = \frac{\cos\delta\cos(\alpha - A)}{\cos D - \eta\sin D} \qquad (2)$$

and since we may re-write (1) in the form

$$d\cos\delta\sin(\alpha - A) + q\sin\delta + r\cos\delta\cos(\alpha - A) = 0 \qquad (3)$$

where

$$d = \cos D_m\sin(A_m - A),\ q = \sin D_m,\ r = \cos D_m\cos(A_m - A) \qquad (4)$$

It may also be written

$$d.\xi + q(\sin D + \eta\cos D) + r(\cos D - \eta\sin D) = 0 \ . \qquad (5)$$

or

$$d.\xi + e.\eta + f = 0 \ . \quad . \quad . \quad . \quad . \qquad (6)$$

where q and r are replaced by e and f such that

$$\left.\begin{aligned} e &= q\cos D - r\sin D = \sin D_m\cos D - \cos D_m\sin D\cos(A_m - A) \\ f &= q\sin D + r\cos D = \sin D_m\sin D + \cos D_m\cos D\cos(A_m - A) \end{aligned}\right\} \qquad (7)$$

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The equation (6) is the equation to the meteor trail on the plate in standard co-ordinates.

5. Now let (ξ_s, η_s) be the standard co-ordinates of any star, and let us measure, in terms of any convenient unit, its distance p_s from the meteor trail. Then

$$k'.p_s\,(d^2+e^2)^{\frac{1}{2}}=d\xi_s+e\eta_s+f$$

where k' is a constant representing the arbitrary scale value; and we can replace $k'\,(d^2+e^2)^{\frac{1}{2}}$ by a new arbitrary constant k^{-1}. Thus

$$p_s=kd.\xi_s+ke.\eta_s+kf$$

From the measures of three different stars for which (ξ_s, η_s) are known we get kd, ke, kf. We can utilise the measures of other stars by any of the well-known processes.

6. Then from (7) we get kq and kr; and from (4) we have

$$\tan\,(\mathrm{A_m}-\mathrm{A})=\frac{d}{r}=\frac{kd}{kr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\tan\,\mathrm{D_m}=\frac{q}{\pm\,(d^2+r^2)^{\frac{1}{2}}}=\frac{kq}{\pm\,(k^2d^2+k^2r^2)^{\frac{1}{2}}} \quad . \quad . \quad (9)$$

which give $\mathrm{A_m}$ and $\mathrm{D_m}$, since A is known.

7. For the trail caught by Mr. Longbottom on August 4, 1907, while he was photographing N.G.C. 6960, with a 12¼-in. reflector of 24-in. focus, exposure 45 minutes; time of meteor $10^{\mathrm{h}}\ 52^{\mathrm{m}}$: it will be a sufficiently good approximation (as the scale is small and the star images large owing to coma and a small driving error) to take as the plate-centre that used for the plate of this region in the Oxford Astrographic Catalogue, viz.—

$$\mathrm{A}=20^{\mathrm{h}}\ 42^{\mathrm{m}} \qquad \mathrm{D}=+30^\circ \qquad (1900{\cdot}0)$$

There was no difficulty in identifying several stars from the Cambridge Catalogue, and a rough reduction from these made it easy to identify on the reflector photograph the other stars shown on the Oxford plate in the neighbourhood. The following measures were then made by Mr. Bellamy:—

OXFORD.

No. in +30°.	Diam.	x	y	p	O−C.
61506	14	7·785	16·391	+1·12	·00
61507	12	8·018	·418	+1·09	·00
61508*	22	8·616	·508	+0·99	·00
61509	14	10·580	·420	+0·91	−·01
61510	20	11·184	·439	+0·83	+·01
61511	22	11·622	·228	+0·92	+·01
61512*	40	11·715	·407	+0·83	−·01
61513	20	12·689	·544	+0·65	+·01
61514	14	12·725	16·360	+0·77	·00

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No. in +30°.	Diam.	x	y	p	O–C.
61542	11	8·444	17·446	+0·44	+·02
61543	12	10·563	410	+0·32	·00
61545	12	10·993	·952	−0·04	·00
61546*	90	11·292	·138	+0·45	−·02
61547	13	15·090	17·401	−0·03	−·02
61580*	35	8·695	18·422	−0·14	−·01
61581	40	9·390	·385	−0·16	−·02
61582	15	9·975	·363	−0·21	·00
61583	20	11·096	·165	−0·17	−·01
61584	12	11·256	·156	−0·18	·00
61585	17	12·179	·631	−0·52	−·02
61586	14	13·643	·877	−0·78	−·01
61587	30	14·341	·929	−0·88	·00
61588	24	15·450	18·888	−0·92	−·02
61606*	30	9·744	19·337	−0·77	·00
61607	12	12·222	·708	−1·17	−·01
61608	12	13·189	·027	−0·84	−·01
61609*	38	14·796	·524	−1·27	+·01
61610	14	15·290	19·659	−1·46	+·08

Measures of 61610 marked as very faint, ? a star.

The values of x and y are not standard co-ordinates but the measures on the Oxford plate; but they can be readily converted into standard co-ordinates by means of the formulæ given at the plate-heading in the Oxford Catalogue. No doubt it is scarcely necessary to measure so many stars, but it seemed possible that there might be systematic error depending on the shape of the star images with a reflector of so short a focus; and thus stars of different magnitudes might give different results.

8. The constants in the linear relation

$$d'x + e'y + f' = p$$

were then found to be as follows:—

$$d' = -{\cdot}0762, \quad e' = -{\cdot}5915, \quad f' = +11{\cdot}416$$

We may omit the last figure as probably not justified by the measures and write

$$d' = -{\cdot}076, \quad e' = -{\cdot}592, \quad f' = +11{\cdot}42$$

With these values the column O – C was formed, and it will be seen that the effect of the brightness of the star is not large.

9. The formulæ for getting standard co-ordinates from x and y are (*see* heading of plate)

$$\xi = x - 13 + {\cdot}00054x - {\cdot}00081y + {\cdot}5124$$
$$\eta = y - 13 + {\cdot}00061x + {\cdot}00043y + {\cdot}1011$$

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Let us write

$$\xi_0 = \xi + 13 - \cdot 51 = \xi + 12\cdot 49$$
$$\eta_0 = \eta + 13 - \cdot 10 = \eta + 12\cdot 90$$

adopting the standard of accuracy found sufficient above.

Then $(x - \xi_0)$, $(y - \eta_0)$ differ by less than ·001 of x and y, and hence we may neglect squares of these differences. Thus

$$x = \xi_0 - \cdot 0005\xi_0 + \cdot 0008\eta_0 = \xi - \cdot 0005\xi + \cdot 0008\eta + 12\cdot 49$$
$$y = \eta_0 - \cdot 0006\xi_0 - \cdot 0004\eta_0 = \eta + \cdot 0006\xi + 0004\eta + 12\cdot 91$$

Hence $p = -\cdot 076\xi - \cdot 592\eta + 11\cdot 42 - \cdot 076 \times 12\cdot 49 - \cdot 592 \times 12\cdot 91$
$\quad = -\cdot 076\xi - \cdot 592\eta + 2\cdot 84$

But ξ η are expressed in reseau intervals, and it was assumed in § 4 that they were in parts of the radius. We must multiply their coefficients by 687·6.

10. Hence as in Eq$^{\text{n}}$. (7)

$$kq \cos \mathrm{D} - kr \sin \mathrm{D} = ke = -\cdot 592 \times 687\cdot 6 = -407\cdot 1$$
$$kq \sin \mathrm{D} + kr \cos \mathrm{D} = kf = +2\cdot 84$$

$$kq = -407 \times \cdot 866 + 2\cdot 84 \times 0\cdot 500 = -351$$
$$kr = +407 \times \cdot 500 + 2\cdot 84 \times 0\cdot 866 = +206$$

and $$\tan(\mathrm{A_m} - \mathrm{A}) = \frac{kd}{kr} = \frac{-\cdot 076 \times 688}{+206} = -\cdot 254$$

$$\therefore \quad \mathrm{A_m} - \mathrm{A} = -14^\circ 15' \text{ or } +165^\circ 45'$$

We shall always get an ambiguity, as there are two poles to the trail. But when we choose one or other of these values, we settle the ambiguity of Eq$^{\text{n}}$. (9), which may be otherwise written

$$\tan \mathrm{D_m} = \frac{kq}{kr} \sec(\mathrm{A_m} - \mathrm{A})$$

$$= \frac{-351}{206} \times 1\cdot 032 = -1\cdot 758 \text{ taking } (\mathrm{A_m} - \mathrm{A}) = -14^\circ 15'$$

$$\mathrm{D_m} = -60^\circ 22'$$

Hence the two poles are for 1900·0

$\mathrm{A_m} = 20^{\mathrm{h}}\ 42^{\mathrm{m}} - 57^{\mathrm{m}}\ 0^{\mathrm{s}}$
$\quad = 19^{\mathrm{h}}\ 45^{\mathrm{m}}\ 0^{\mathrm{s}}$
$\mathrm{D_m} = -60^\circ\ 22'$

and

$\mathrm{A_m} = 7^{\mathrm{h}}\ 45^{\mathrm{m}}\ 0^{\mathrm{s}}$
$\mathrm{D_m} = +60^\circ\ 22'$

They may be brought up to date by well known rules.